Hard Diffraction in the QCD Dipole Picture

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Abstract

Using the QCD dipole picture of the BFKL pomeron, the gluon contribution to the cross-section for single diffractive dissociation in deep-inelastic high-energy scattering is calculated. The resulting contribution to the proton diffractive structure function integrated over t is given in terms of relevant variables, $x_{\mathcal{P}}, Q^2$, and $\beta = x_{Bj}/x_{\mathcal{P}}$. It factorizes into an explicit $x_{\mathcal{P}}-$dependent Hard Pomeron flux factor and structure function. The flux factor is found to have substantial logarithmic corrections which may account for the recent measurements of the Pomeron intercept in this process. The triple Pomeron coupling is shown to be strongly enhanced by the resummation of leading logs. The obtained pattern of scaling violation at small β is similar to that for F_2 at small x_{Bj}.

1. Recently, new measurements of the proton diffractive structure function at small x and very large Q^2 were presented by H1 and ZEUS experiments [1, 2]. The observed 3-dimensional structure function factorizes:

$$F_2^{D(3)}(x_{Bj}, Q^2, x_{\mathcal{P}}) = f(x_{\mathcal{P}}) \, F_2^{D(2)}(\beta, Q^2) \ . \tag{1}$$

Here $\beta \equiv Q^2/(Q^2 + M^2)$, $x_{\mathcal{P}} = x_{Bj}/\beta$, and M^2 is the mass of the diffractively excited system. This factorized form is naturally interpreted as the product

of a Pomeron flux factor inside the proton and its corresponding structure function[3]. However, other theoretical interpretations are also possible[4].

The aim of the present paper is to investigate the perturbative QCD contribution to the process in question using the colour dipole approach [5, 6], which is known [5, 6, 7] to reproduce the physics of the "Hard Pomeron"[8]. We calculate the diffraction dissociation of the virtual photon on the proton at small x_{Bj} and large Q^2. Our main result is the explicit formula for the small $x_{\mathcal{P}}$ behaviour of the 3-dimensional diffractive structure function

$$F_2^{D(3)}(Q^2, x_{\mathcal{P}}, \beta) \equiv \int dt F_2^{D(4)}(Q^2, x_{\mathcal{P}}, \beta, t) =$$

$$= \frac{2e_f^2 \alpha^5 N_c^2}{\pi^2} \left(\frac{2a(x_{\mathcal{P}})}{\pi}\right)^3 x_{\mathcal{P}}^{-1-2\Delta_{\mathcal{P}}} \int_{c-i\infty}^{c+i\infty} \frac{d\gamma}{2\pi i} \left(\frac{r_0 Q}{2}\right)^\gamma H(\gamma) \, \beta^{-\alpha N_c \chi(\gamma)/\pi} \quad (2)$$

where $H(\gamma)$ is given[1] by

$$H(\gamma) = V(\gamma) \frac{4}{\gamma^2 (2-\gamma)^4} \frac{\Gamma(3-\frac{\gamma}{2})\Gamma^3(2-\frac{\gamma}{2})\Gamma(2+\frac{\gamma}{2})\Gamma(1+\frac{\gamma}{2})}{\Gamma(4-\gamma)\Gamma(2+\gamma)} \quad (3)$$

with

$$V(\gamma) = \int_0^1 F(1-\frac{\gamma}{2}, 1-\frac{\gamma}{2}; 1; y^2) \, dy \quad (4)$$

(F is the hypergeometric function). $\chi(\gamma)$ is the eigenvalue of the BFKL kernel defined as

$$\chi(\gamma) = 2\psi(1) - \psi(1-\frac{\gamma}{2}) - \psi(\frac{\gamma}{2}), \quad (5)$$

and $a(\xi)$ is given by

$$a(\xi) = [7\alpha N_c \zeta(3) \log(1/\xi)/\pi]^{-1}. \quad (6)$$

$\Delta_{\mathcal{P}} \equiv \alpha_{\mathcal{P}} - 1 = \frac{\alpha N_c}{\pi} \chi(1)$. r_0 is a non-perturbative parameter (defined by (30) as the average of the transverse dipole size inside the target) which cannot be determined within the present approach. Finally, e_f^2 is the sum of the squares of quark charges.

The formula (2) has several interesting features.

(i) *Factorization*. One sees that $F_2^{D(3)}$ is a product of two factors: one depends only on $x_{\mathcal{P}}$, another one depends on β and Q^2. The first one can

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[1]We consider here only the transverse photon contribution.

thus be interpreted as the "Pomeron flux factor" and the second one as the "Pomeron structure function" [1, 2].

(ii) *Pomeron flux factor.* With this identificaton, one obtains for the Pomeron flux factor

$$\Phi_{\mathcal{P}} = C_{\Phi} \, x_{\mathcal{P}}^{-1-2\Delta_{\mathcal{P}}} \left(\frac{2a(x_{\mathcal{P}})}{\pi} \right)^3 \tag{7}$$

where C_{Φ} is an arbitrary constant. One sees that the obtained $x_{\mathcal{P}}$ dependence differs from the normally assumed power law $x_{\mathcal{P}}^{-1-2\Delta_{\mathcal{P}}}$ by a logarithmic factor $(\log(1/x_{\mathcal{P}}))^{-3}$, as found already in the triple Pomeron limit [7]. Our calculation shows that this logarithmic correction is rather general and also applies beyond this limit. It would be of course very interesting to verify this prediction with the data. In this context we note that, when fitted with a power law, the formula (7) gives an effective Pomeron intercept

$$\Delta_{\mathcal{P}}^{eff} = \Delta_{\mathcal{P}} - \frac{3}{2 log(1/x_{\mathcal{P}})}. \tag{8}$$

We would like to emphasize that this correction is rather substantial even at rather small $x_{\mathcal{P}}$ (at $x_{\mathcal{P}} = 10^{-3}, \Delta_{\mathcal{P}}^{eff} - \Delta_{\mathcal{P}} \approx -0.2$). One sees that this effect agrees both in sign and in magnitude with the difference between the Pomeron intercepts observed in F_2 [9, 10] and in the diffractive structure function $F_2^{D(3)}$ [1, 2] and may thus be a simple explanation of the apparent contradiction between these two measurements. It would clearly be of great interest to analyze the future data using the form (7).

(iii) *The Pomeron structure function* depends explicitly on Q^2 and thus the model predicts violation of scaling. To have a feeling on the pattern of this scaling violation and also on the dependence of $F_{\mathcal{P}}$ on β, it is illuminating to evaluate the integral in (2) by the saddle-point method. The result is

$$F_{\mathcal{P}}(Q^2, \beta) = \frac{e_f^2 \alpha^5 N_c^2}{C_{\Phi} \pi^2} \left(\frac{r_0 Q}{2} \right)^{\gamma_0} H(\gamma_0) \left(\frac{2a(\beta) 7\zeta(3)}{\pi \chi''(\gamma_0)} \right)^{-\frac{1}{2}} \beta^{-\alpha N_c \chi(\gamma_0)/\pi}, \tag{9}$$

γ_0 being determined from the saddle-point equation:

$$\frac{\alpha N_c}{\pi} \chi'(\gamma_0) \, \log \beta = \log(\frac{Q r_0}{2}). \tag{10}$$

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In the interesting "triple Pomeron" limit, $\beta \approx 0, \gamma_0 \approx 1$, (9) simplifies into

$$F_{\mathcal{P}}(Q^2, \beta \approx 0) = \frac{e_f^2 \alpha^5 N_c^2}{C_\Phi \pi^2} H(1) \frac{r_0 Q}{2} \beta^{-\Delta_{\mathcal{P}}} \left(\frac{2a(\beta)}{\pi} \right)^{\frac{1}{2}} \exp[-\frac{a(\beta)}{2} log^2(\frac{r_0 Q}{2})].$$

(11)

This formula gives a pattern of scaling violation typical of the exchange of a hard pomeron [11].

(iv) *Triple Pomeron coupling.* $H(\gamma)$ gives the triple pomeron coupling. $H(1)$ can be explicitly evaluated from (3) with the result $H(1) = 9\pi^2 G/64$ where $G = .915...$ is Catalan's constant. This value can be compared with the corresponding coupling obtained from the same expressions using the expansion $\gamma \to 0$, which corresponds to the first-order perturbative QCD result [12]. One finds $H^{pert}(\gamma) = \gamma^{-3}/12$. One may observe a large enhancement factor $H(1)/H^{pert}(1) = \frac{27\pi^2 G}{8} \approx 30$ due to the leading $log(1/x)$ resummation which is taken into account in the QCD dipole model. It provides a theoretical hint for the surprisingly large experimentally observed hard-diffractive cross-section.

2. We shall now outline the derivation of the Eq.(2). In order to formulate the problem of diffraction dissociation we use the old idea of Good and Walker (see e.g.[13, 14]), i.e. the expansion of the initial colliding state in the diag onal basis of the eigenstates of absorption. To this end, we observe that the dipole representation corresponds precisely to such a decomposition. Indeed, the amplitude for elastic scattering of two dipoles of transverse size $x_1, x_2,$, is simply given by two-gluon exchange[7, 15], namely:

$$T(x_1, x_2) = 4\pi \alpha^2 \int \frac{dl}{l^3} [1 - J_0(lx_1)][1 - J_0(lx_2)] ,$$

(12)

and this interaction changes neither their transverse size and position nor their rapidities. Using this general framework we write the cross-section for single diffractive dissociation [2] of a virtual photon on a proton as:

$$\frac{\beta d\sigma}{d\beta d^2 b} = \int d^2 \bar{r} d\bar{z} \, \bar{\Phi}(\bar{r}, \bar{z}; Q^2) \, \sigma_d(\bar{r}, b, \beta, x_{\mathcal{P}})$$

(13)

where $\bar{\Phi}$ is the probability of the virtual photon to fluctuate into a $q\bar{q}$ pair and σ_d is the single diffractive cross-section in dipole-proton scattering

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[2]An analogous formulation was used in [7].

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$$\sigma_d = \int \frac{dx}{x} d^2\bar{s}_1 \; \hat{\rho}_1(b + \bar{s}_1, x, \xi) \int \frac{dx'}{x'} d^2\bar{s}_2 \; \hat{\rho}_1(b + \bar{s}_2, x', \xi)$$

$$\int \frac{d\bar{x}_1}{\bar{x}_1} \frac{d\bar{x}_2}{\bar{x}_2} \; \rho_2(\bar{r}; \bar{s}_1, \bar{x}_1; \bar{s}_2, \bar{x}_2; x_{Bj}/\xi, x_\mathcal{P}/\xi) \; T(x, \bar{x}_1) \; T(x', \bar{x}_2) \; . \tag{14}$$

ρ_2 is the double dipole density in the colliding dipole of transverse size $\bar{r}$ [7, 15] and $\hat{\rho}_1$ is the single dipole density in the proton:

$$\hat{\rho}_1(b, x, \xi) = \int d^2r dz \Phi(r, z) \rho_1(r, b, x, \xi) \tag{15}$$

where Φ is the square of the proton wave function and ρ_1 is the single dipole density in a dipole of transverse size r [15]

$$\rho_1(r, b, x, \xi) = \frac{r}{4xb^2} (2a(\xi)/\pi)^{\frac{3}{2}} \log\left(b^2/rx\right) \xi^{-\Delta_\mathcal{P}} e^{-\frac{a(\xi)}{2}\log^2\left(b^2/rx\right)} \; . \tag{16}$$

We start by computing the Mellin transform

$$\tilde{\sigma}_d(\gamma; b; \beta, x_\mathcal{P}) \equiv \int d\bar{r} \; \bar{r}^{\gamma-1} \sigma_d(\bar{r}; b; \beta, x_\mathcal{P})/\bar{r}^2. \tag{17}$$

Using the methods developed in [16], one arrives at the following formula for the Mellin transform of the double-dipole density ρ_2:

$$\tilde{\rho}_2(\gamma; s_1, x_1, s_2, x_2; \xi_1, \xi_2) = \frac{2\alpha N_c}{\pi} \left(\frac{\xi_2}{\xi_1}\right)^{\frac{\alpha N_c}{\pi}\chi(\gamma)}$$

$$\int \frac{dr_1}{r_1} \frac{dr_2}{r_2} \rho_1(r_1, b_1, x_1, \xi_2) \; \rho_1(r_2, b_2, x_2, \xi_2) \; W(r_1, r_2) \tag{18}$$

where, for $r_1 < r_2$, the symmetric function $W(r_1, r_2)$ is given by [16]:

$$W(r_1, r_2) = r_2^{\gamma-2} \; F(1 - \gamma/2, 1 - \gamma/2; 1; (r_1/r_2)^2) \tag{19}$$

By repeated application of the formula [15]

$$\int \frac{dx_1}{x_1} \frac{dx_2}{x_2} T(x_1, x_2) d^2s \; \rho_1(r_1, s, x_1, \xi_1) \rho_1(r_2, b + s, x_2, \xi_2) =$$

$$\pi \alpha^2 \frac{r_1 r_2}{b^2} (\xi_1 \xi_2)^{-\Delta_\mathcal{P}} \left(\frac{2a(\xi_1 \xi_2)}{\pi}\right)^{3/2} \log\left(\frac{b^2}{r_1 r_2}\right) \exp[-\frac{a(\xi_1 \xi_2)}{2} \log^2\left(\frac{b^2}{r_1 r_2}\right)] \tag{20}$$

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one obtains

$$\tilde{\sigma}_d = \frac{\pi\alpha^5 N_c}{b^4 x_{\mathcal{P}}^{\Delta_{\mathcal{P}}}} \beta^{-\alpha N_c \chi(\gamma)/\pi} \left(\frac{2a(x_{\mathcal{P}})}{\pi}\right)^3 \int dx_{12} dx_{02} W(x_{12}, x_{02}) D(x_{12}) D(x_{02}) \tag{21}$$

where

$$D(x) = \int d^2 r dz \Phi(r,z) r ln \frac{b^2}{rx} \exp[-\frac{a(x_{\mathcal{P}})}{2} ln^2 \frac{b^2}{rx}]. \tag{22}$$

Using (19) it is possible to perform the integrals in (21) and obtain the following expression for the Mellin transform of σ_d

$$\tilde{\sigma}_d = 32\pi\alpha^5 N_c < x^2(b, x_{\mathcal{P}}) >^2 \frac{1}{b^{2-\gamma}} \frac{V(\gamma)}{\gamma} \beta^{-\alpha N_c \chi(\gamma)/\pi} \tag{23}$$

where

$$< x^2(b, x_{\mathcal{P}}) >= \int d^2 r dz \Phi(r,z) \int \frac{dx}{x} x^2 \rho_1(r, b, x, x_{\mathcal{P}}) =$$

$$= \frac{1}{4b} x_{\mathcal{P}}^{-\Delta_P} (\frac{2a(x_{\mathcal{P}})}{\pi})^{\frac{3}{2}} \int d^2 r dz \Phi(r,z) r \log(\frac{b}{r}) \exp[-\frac{a(x_{\mathcal{P}})}{2} \log^2(\frac{b}{r})]. \tag{24}$$

As seen from (24), the dimensionless quantity $< x^2 >$ can be interpreted as the density distribution for the average of the square of the transverse sizes of the dipoles inside the proton at a fixed impact parameter b. It summarizes the whole information about the proton wave function which is relevant for the process we consider.

To obtain σ_d from (23) one has to perform the inverse Mellin transform

$$\sigma_d(\bar{r}; b, ; \beta, x_{\mathcal{P}}) = \int_{c-i\infty}^{c+i\infty} \frac{d\gamma}{2\pi i} \bar{r}^{2-\gamma} \tilde{\sigma}_d(\gamma; b; \beta, x_{\mathcal{P}}) \tag{25}$$

with $c > 0$. Inserting (23) and (25) into (13) and using [6] (we neglect quark masses and the longitudinal cross-section)

$$\bar{\Phi}(r, z; Q^2) = \frac{N_c \alpha_{em}}{(2\pi)^2} e_f^2 (z^2 + (1-z)^2) \hat{Q}^2 K_1(\hat{Q}r) \tag{26}$$

with $\hat{Q}^2 = z(1-z)Q^2$ one can perform the integrations over z and r. The result is

$$\frac{\beta d\sigma}{d\beta d^2 b} = \frac{32\alpha_{em}}{\pi} e_f^2 \alpha^5 N_c^2 < x^2(b, x_{\mathcal{P}}) >^2 (bQ)^{-2}$$

$$\int_{c-i\infty}^{c+i\infty} \frac{d\gamma}{2\pi i} (\frac{bQ}{2})^\gamma (2-\gamma)^3 H(\gamma) \beta^{-\alpha N_c \chi(\gamma)/\pi} \tag{27}$$

Eq.(27) represents our final result for the diffractive photon-nucleon cross-section at fixed impact parameter.

The diffractive structure function as defined, e.g., in [1, 2] is obtained from (27) using the relation

$$F_2^{D(4)}(Q^2, x_{\mathcal{P}}, \beta, b) = \frac{Q^2}{4\pi^2 \alpha_{em}} x_{\mathcal{P}}^{-1} \frac{\beta d\sigma}{d\beta d^2 b}. \tag{28}$$

Since b-dependence of the diffractive structure function is not experimentally accessible, in the following we consider its integral over $d^2 b$ which is obviously equal to the integral over the whole range of the momentum transfer t to the target proton and thus measurable [3]. To integrate (27) over $d^2 b$, however, it is necessary to know the form of $< x^2(b, x_{\mathcal{P}}) >$ and thus the form of the proton wave function $\Phi(r, z)$. Fortunately, in the limit $x_{\mathcal{P}} \to 0$, and $b \gg r$, $< x^2 >$ is not very sensitive to this input and can be approximated as

$$< x^2(b, x_{calP}) > = \frac{r_0}{4b} x_{\mathcal{P}}^{-\Delta_P} \left(\frac{2a(x_{\mathcal{P}})}{\pi}\right)^{\frac{3}{2}} \log(\frac{b}{r_0}) \exp[-\frac{a(x_{\mathcal{P}})}{2} \log^2(\frac{b}{r_0})]. \tag{29}$$

where

$$r_0 = \int d^2 r dz \Phi(r, z) r. \tag{30}$$

Using (29) one can integrate (27) over b. Taking into account (28) we obtain (2).

3. To summarize, using the QCD dipole framework we have calculated the large mass contribution to the process of diffraction dissociation of the virtual photon at large Q^2 in the limit of very small $x_{\mathcal{P}}$. We find that the diffractive structure function, when integrated over t, takes a particularly simple factorizable form [4]. However, the resulting "Pomeron flux factor" is modified by logarithmic corrections which lead to an effective intercept substantially lower than the one obtained from the proton structure function. This should have clear experimental consequences.

Our calculation provides an explicit formula for the triple (hard-)pomeron coupling. In this context we find a rather large asymptotic (i.e. for $\gamma = 1$)

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[3]For structure functions, the relation between b-dependence and t-dependence is not simple and goes beyond the scope of this work.

[4]This is in contrast with the rather complicated behaviour at small t found in [17].

enhancement factor as compared to the first order perturbative calculation. This means that the Lipatov resummation is even more important here than in the total cross-section [12].

Our results imply that the pattern of scaling violation at small β should be similar to that observed for total cross section at small x_{Bj}. It is important to realize, however, that this conclusion does not apply for large values of β (i.e. finite ratio M^2/Q^2). In this region of β the valence $q\bar{q}$ content of the pomeron should be taken into account (see, e.g., [18] and references quoted there). This goes beyond the scope of the present investigation.

Acknowledgments

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